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# Numerical Optimization of an Annular Field Reversed Configuration Translation Experiment

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Annular field reversed configuration (AFRC) devices form annular plasma toroids between a pair of concentric cylindrical coils. This plasmoid remains detached from the external magnetic field so that it can be ejected from the coils, making AFRCs viable pulsed inductive plasma accelerators. Though numerous formation studies on AFRCs are available, no successful translation studies have been published. Michigan Technological University, in conjunction with the Air Force Research Laboratory, is investigating the translation of AFRCs as pulsed inductive plasma accelerators. The first step in this investigation is to develop an annular electromagnetic launcher model to study the basic translation characteristics of the device. The launcher model treats the plasmoid as a rigid conducting slug, accelerated out of the coils by a Lorentz force. It predicts coil and plasmoid currents, plasmoid trajectories, and acceleration efficiencies for various input conditions. The model has been optimized for peak acceleration efficiency using a combination of non-dimensional analysis, genetic algorithms, and gradient-based numerical optimization routines. A description of the model, explanation of the numerical optimization techniques, and preliminary results from the model are presented in this paper.

## Nomenclature

$\eta_{accel}^*$	Acceleration efficiency [non-dimensional]	$m$	Plasmoid mass
$\eta_{accel}$	Acceleration efficiency	$M_{13}$	Mutual inductance (Outer coil-plasmoid)
$C_1$	Circuit capacitance	$M_{23}$	Mutual inductance (Inner coil-plasmoid)
$I_1^*$	Outer coil current [non-dimensional]	$n_i$	Plasma density
$I_2^*$	Inner coil current [non-dimensional]	$n_{13}$	Exponential power (Outer coil-plasmoid)
$I_3^*$	Plasmoid current [non-dimensional]	$n_{23}$	Exponential power (Inner coil-plasmoid)
$I_0$	Coil circuit current	$R_1$	Outer coil circuit external resistance
$I_1$	Outer coil circuit current	$R_2$	Inner coil circuit external resistance
$I_2$	Inner coil circuit current	$R_3$	Plasmoid resistance
$I_3$	Plasma circuit current	$R_e$	External circuit resistance
$k_{13}$	Outer coil-plasmoid coupling coefficient	$t$	Time
$k_{23}$	Inner coil-plasmoid coupling coefficient	$t^*$	Time [non-dimensional]
$L_1$	Outer coil inductance	$T_e$	Electron temperature [eV]
$L_2$	Inner coil inductance	$u_z^*$	Plasmoid velocity [non-dimensional]
$L_3$	Plasmoid inductance	$u_f^*$	Final plasmoid velocity [non-dimensional]
$L_e$	External circuit inductance	$u_f$	Final plasmoid velocity
$L_{e1}$	Outer coil circuit external inductance	$u_z$	Plasmoid velocity
$L_{e2}$	Inner coil circuit external inductance	$V_c^*$	Capacitor voltage [non-dimensional]
		$V_0$	Capacitor charging voltage
		$V_c$	Capacitor voltage

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$z$	Plasmoid position	$z_{23}$	Coupling scale length (Inner coil-plasmoid)
$z^*$	Plasmoid position [non-dimensional]	$z_{s13}$	Profile z shift (Outer coil-plasmoid)
$z_{13}$	Coupling scale length (Outer coil-plasmoid)	$z_{s23}$	Profile z shift (Inner coil-plasmoid)

## I. Introduction

ANNULAR field reversed configuration (AFRC) devices offer significant potential as high power electric propulsion technology. They form robust, high-energy density compact plasma toroids at relatively low energies ( $\sim 250$  J). These plasmoids have a completely self-contained magnetic field structure which allows them to be expelled from their formation chamber at high velocity, similar to the pulsed inductive thruster<sup>1</sup>(PIT) and conical theta pinch thruster.<sup>2</sup> Michigan Technological University (MTU), in conjunction with the Air Force Research Laboratory (AFRL), is constructing an experiment to study the behavior of translating AFRCs. This experiment, the TeXOCOT, is a next-generation device of AFRL/University of Michigan's XOCOT experiment<sup>3</sup> and AFRL's XOCOT-T experiment.<sup>4</sup>

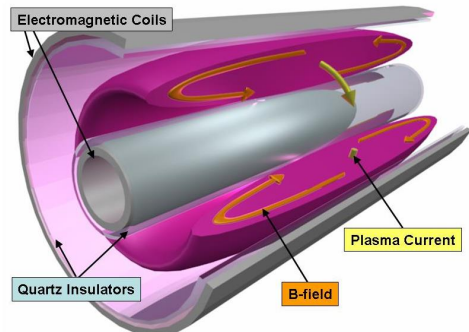
The TeXOCOT program intends to measure the translation properties of an AFRC plasmoid, including its velocity, momentum, and acceleration efficiency. A new AFRC translation experiment for taking these measurements will be constructed in MTU's Ion Space Propulsion Lab. This experiment is being designed with the aid of an annular electromagnetic launcher model to identify a set of operating conditions, such as cone angle, circuit charging voltage, plasmoid mass, etc., for which translation of the plasmoid is possible and acceleration efficiency of the plasmoid is maximized. This paper presents the annular electromagnetic launcher model and optimization techniques used on the model to find the peak acceleration efficiency of an AFRC plasmoid. Preliminary findings from these methods indicate promising results, though further refinement is necessary.

## II. Background: Annular Field Reversed Configurations

An annular field reversed configuration is a compact plasma toroid formed in the annular region between two coaxial coils, as shown in Figure 1. The purely poloidal B-field is supported by an induced toroidal (azimuthal) current. This diamagnetic current forms a closed magnetic field to confine the plasma. AFRCs were first proposed by Phillips<sup>5</sup> as a low-voltage formation method for fusion plasma, but subsequent research at the University of Washington on the Coaxial Slow Source (CSS)<sup>6</sup> showed temperatures and densities in an AFRC are insufficient for fusion.

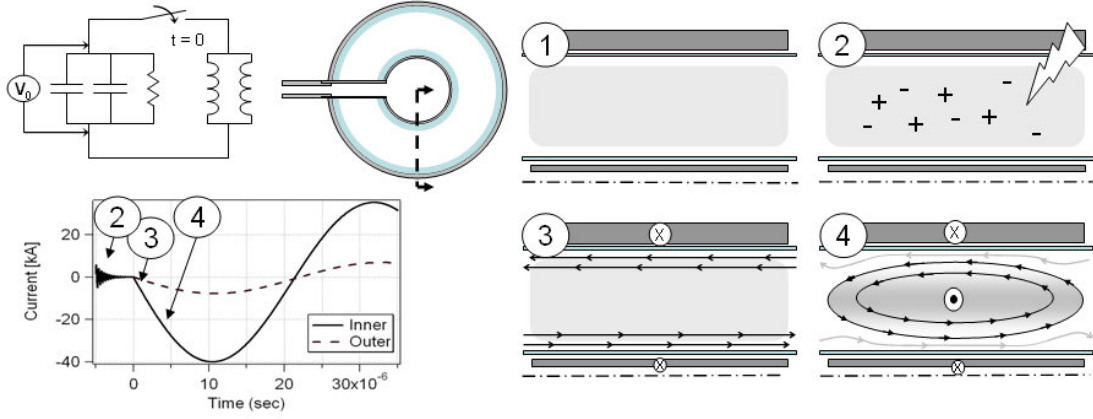
Formation of AFRCs remains an empirical process which is not well-understood, despite numerous studies.<sup>3,7,8</sup> The plasma is turbulent fluid, subject to complex magnetohydrodynamic instabilities requiring numerical plasma studies to resolve. Despite the limitation, the formation of AFRCs can be illustrated by considering basic electromagnetic principles.

Annular FRCs can either be formed with the coils connected in parallel or the coils operating independently from one another. The parallel formation method will be the subject of this work; a dissertation by Smith<sup>9</sup> contains a detailed description of the independent coil mode. Figure 2 displays the circuit configuration, typical current waveforms, and the formation sequence for the parallel coil operation. The channel between the coils is filled with an inert gas (Step 1), which is then partially ionized or pre-ionized (Step 2). Upon completion of the pre-ionization stage, the capacitors are discharged into the coils adding oppositely directed magnetic fields in the annulus between the coils (Step 3). As a consequence of Faraday's Law, the rising currents in the coils cause a current to develop in the plasma, opposing this change in field (Step 4). This current is diamagnetic so that it creates a closed **B**-field topology to confine the plasma. Further increasing the currents in the coils causes the current in the plasma to increase, ionizing and heating it. External magnetic pressure from both coils balances with the plasma pressure to help keep the toroid well-centered in the annulus. Confinement of the plasma is improved if oppositely directed field lines near the coil ends are allowed to tear and reconnect, though evidence of this



**Figure 1. AFRCs are formed inductively between two coaxial coils as shown.**

remains unclear.



**Figure 2. Parallel coil configuration showing circuit diagrams, schematics, and typical current waveforms. The formation sequence starts with a neutral backfill (1) followed by a pre-ionization of the fill gas (2). Current is pulsed through the coils (3) and the plasma current develops to oppose the change in B-field, resulting in a closed confinement (4).**

Successful translation studies on AFRCs are not readily available. The XOCOT-T experiment attempted to translate the plasmoid, however evidence of translation was not apparent. In principle, AFRCs can be translated the same way as their close relative, the FRC.<sup>10</sup> Traditional FRCs do not use a center coil and operate on much faster ( $\sim 5 \mu\text{s}$ ) timescales. Most FRC devices translate the plasmoid using a conical outer coil in place of the traditional cylindrical coil. This creates a small radial magnetic field component which crosses with the azimuthal plasma current to create an axially directed Lorentz force. This method has been used successfully in the FRX/C-T,<sup>11</sup> the MTF experiment,<sup>12</sup> and was the translation method for the XOCOT-T.<sup>4</sup>

The Lorentz force acting on the plasmoid must be sufficiently large to overcome the plasmoid inertia and it must accelerate the plasmoid in time comparable to the lifetime of the configuration. For AFRCs, the configuration typically lasts only about 20-30  $\mu\text{s}$ .<sup>6</sup> Once translation of the plasmoid is initiated, the plasmoid can only be expelled from the formation chamber if it is completely detached from the magnetic field of the coil. Since the magnetic field null is along the centerline of the annulus in the parallel coil configuration, the plasmoid should always remain detached from the coils.

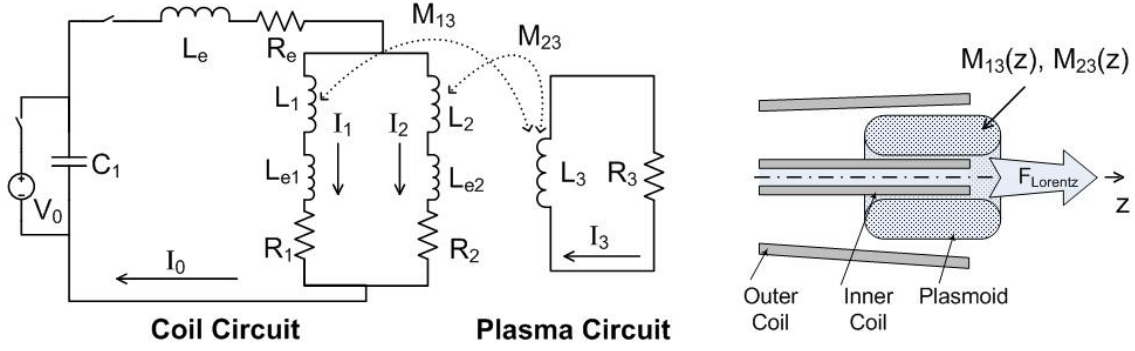
### III. Annular Electromagnetic Launcher Model

Dynamic circuit models are a popular method for predicting the translation physics of a pulsed inductive plasmoid accelerators<sup>1,13,14,15</sup> and electromagnetic launchers.<sup>16</sup> These models couple the circuit equations with Newton's law, producing a set of simultaneous equations to describe the coupled electrical and dynamic behavior of the system. In short, they simplify the device down to a collection of circuit elements which can be solved using first order ordinary differential equations rather than computationally intense particle codes.

Similar to other models, the annular electromagnetic launcher model treats the plasmoid as a rigid conducting slug accelerated by a Lorentz force. This simplification masks the AFRC formation where the plasmoid is changing in shape as external magnetic field pressure balances with internal plasma pressure. Instead, it assumes that equilibrium is reached and the plasmoid maintains a constant shape. The model also neglects the expansion that occurs when the plasmoid leaves the coils, since this expansion for an AFRC has not been investigated.

The AFRC plasmoid forms inductively and is magnetically coupled to the coils through mutual inductance, which decays as it leaves the coils. The mutual inductance controls the Lorentz force by determining the interaction of the plasma current and the coil current. The currents can be calculated using circuit diagrams. Figure 3 provides a pictorial overview of the annular electromagnetic launcher model, operating in the parallel coil mode.

The equations of motion for the plasmoid can be derived by calculating the Lorentz force from  $\mathbf{J} \times \mathbf{B}$ ,



**Figure 3. Schematic of the annular electromagnetic launcher model. The plasmoid is depicted with circuit elements  $L_3$  and  $R_3$ , magnetically coupled to the coils through  $M_{13}$  and  $M_{23}$ .**

but this requires solving several intense integrals. A much easier and equivalent description can be found from conservation of energy, considering how the energy of the system changes as the plasmoid travels out of the coils. Magnetic field energy will change as the mutual inductance changes. This change in magnetic field energy can be related to Newton's 2nd Law through the work-kinetic energy theorem ( $F_z = -\delta U/\delta z$ ), giving:

$$m \frac{du_z}{dt} = -\frac{dM_{13}}{dz} I_1 I_3 - \frac{dM_{23}}{dz} I_2 I_3 \quad (1)$$

where

$$u_z = \frac{dz}{dt} \quad (2)$$

In this model, position of the plasmoid  $z$  is tracked with regard to the trailing edge and  $z = 0$  occurs at the small cone end of the coils. Computing  $I_1$ ,  $I_2$ , and  $I_3$  requires a circuit analysis of the system, yielding the following equations:

$$V_C(t) = L_e \frac{dI_0}{dt} + R_e I_0 + (L_1 + L_{e1}) \frac{dI_1}{dt} + R_1 I_1 - I_3 \frac{dM_{13}}{dz} u_z - M_{13} \frac{dI_3}{dt} \quad (3)$$

$$I_0 = I_1 + I_2 \quad (4)$$

$$0 = (L_1 + L_{e1}) \frac{dI_1}{dt} + R_1 I_1 + (M_{23} - M_{13}) \frac{dI_3}{dt} + \left( \frac{dM_{23}}{dz} - \frac{dM_{13}}{dz} \right) u_z I_3 - (L_2 + L_{e2}) \frac{dI_2}{dt} - R_2 I_2 \quad (5)$$

$$0 = R_3 I_3 + L_3 \frac{dI_3}{dt} - M_{13} \frac{dI_1}{dt} - I_1 \frac{dM_{13}}{dz} u_z - I_2 \frac{dM_{23}}{dz} u_z - M_{23} \frac{dI_2}{dt} \quad (6)$$

$$\frac{dV_C}{dt} = -\frac{I_0}{C_1} \quad (7)$$

The time derivatives of  $M_{13}$  and  $M_{23}$  in Equations 3, 5, and 6 have been separated into a positional derivative and velocity using the chain rule. Their profiles  $M_{13}(z)$  and  $M_{23}(z)$  as well as the coil and plasmoid inductances  $L_1$ ,  $L_2$ , and  $L_3$  are determined from the coil and plasmoid geometry. Given the coil radii, lengths, cone angle, and plasmoid geometry, the mutual inductance profiles  $M_{13}(z)$  and  $M_{23}(z)$  and  $L_1$ ,

$L_2$ ,  $L_3$  can be calculated using analytical equations, experimental methods, or electromagnetic field solvers. As multiple authors have discovered<sup>1,14,17</sup>, the  $M(z)$  profile satisfies an exponential distribution, similar to:

$$M(z) = k_0 \sqrt{L_p L_c} \exp \left( - \left( \frac{|z - z_{shift}|}{z_{scale}} \right)^n \right) \quad (8)$$

where  $k_0$  is the coupling coefficient,  $L_p$  and  $L_c$  are the plasmoid and coil inductances ( $L_3$  and  $L_1$  or  $L_2$ ),  $z_{scale}$  is the coupling scale length,  $n$  is the exponential power term, and  $z_{shift}$  is the location of maximum  $M(z)$ . The difference in geometry between the outer coil-plasmoid combination and the inner coil-plasmoid combination will result in different coupling for both systems, meaning that for each geometry combination, the coefficients  $k_0$ ,  $z_{scale}$ ,  $z_{shift}$ ,  $n$  will have unique values ( $k_{13}$ ,  $k_{23}$ ,  $z_{13}$ ,  $z_{23}$ ,  $z_{s13}$ ,  $z_{s23}$ ,  $n_{13}$ ,  $n_{23}$ ). It has been found that for large cone angles (greater than  $10^\circ$ ),  $n$  approaches 1 and  $z_{shift}$  is zero. For small cone angles,  $n$  and  $z_{shift}$  increase with decreasing cone angle.

Plasmoid resistance  $R_3$  and plasmoid mass  $m$  can be estimated by assuming a fully ionized plasma with uniform properties of density  $n_i$  and temperature  $T_e$ . Resistivity then arises as a result of Coloumb collisions of the current carrying electrons with stationary ions. Spitzer<sup>18</sup> reports this resistivity for a diamagnetic plasma (like that in an AFRC) in terms of the ion charge number  $Z$ , the electron temperature (in eV)  $T_e$ , and the Coloumb logarithm  $\ln \Lambda$ :

$$\rho_{Spitzer} = 1.03 \times 10^{-4} \frac{Z \ln \Lambda}{T_e^{3/2}} \quad (9)$$

For an AFRC,  $\ln \Lambda$  of 12-15 can be used. Using the estimated resistivity, plasma resistance can be calculated from  $R = \rho l / A$ , where  $l$  is the plasmoid length and  $A$  is the cross sectional area. The plasmoid mass, using the assumption of a uniform density, is then estimated as:  $m = n_i V M_m / N_A$ , where  $V$  is the plasmoid volume,  $M_m$  is the molar mass of the plasmoid gas,  $N_A$  is Avogadro's number.

Acceleration efficiency of the plasmoid can be defined as the final plasmoid kinetic energy compared to the energy initially stored in the capacitor bank:

$$\eta_{accel} = \frac{m u_f^2}{C_1 V_0^2} \quad (10)$$

In this work, acceleration efficiency is the figure of merit; optimization of the model will focus on finding its maximum.

The launcher model is an initial value problem, requiring the values for  $I_1$ ,  $I_2$ ,  $I_3$ ,  $V_c$ ,  $u_z$ , and  $z$  to be specified at  $t = 0$ . These 6 initial conditions, combined with the 7 independent circuit elements ( $C_1$ ,  $L_e$ ,  $R_e$ ,  $L_{e1}$ ,  $L_{e2}$ ,  $R_1$ ,  $R_2$ ), 4 coil geometry parameters (outer coil radius, outer coil cone angle, inner coil radius, coil length), 2 plasma geometry parameters (height, width), and 2 plasma properties ( $n_i$ ,  $T_e$ ), require that 21 inputs be specified for the model. This large parameter set takes 5 minutes to solve on a single-core processor, with the bulk of the time consumed calculating  $M_{13}(z)$  and  $M_{23}(z)$ .

## IV. Methodology

The long computation time and large number of inputs make optimization of the annular electromagnetic launcher model challenging. Reduction of the computational overhead can be achieved by removing the geometry dependence from the model and optimizing  $L_1$ ,  $L_2$ ,  $L_3$ ,  $M_{13}(z)$ , and  $M_{23}(z)$  independently. When these optimal parameters are found, a coil and plasmoid geometry can be fit to them. This method, however, does not ensure that the optimal  $L_1$ ,  $L_2$ ,  $L_3$ ,  $M_{13}(z)$ , and  $M_{23}(z)$  can produce a valid geometrical solution. To increase the likelihood of a valid output, the model equations can be non-dimensionalized so that scaling relations between parameters are developed. Optimization of the scaling relations results in generalized solutions to which may satisfy several geometries. Also, if the constraints on the output of the optimization routine are relaxed so that it locates several acceptable extrema rather than producing a single output, the chance of finding a valid geometrical solution is further increased.

These ideas were implemented in this work. First, the model equations in Section III were non-dimensionalized to put the solutions in terms of generalized scaling relations. This also brought the solution time for a single set of parameters down to a few seconds. Next, the non-dimensional equations were optimized for peak acceleration efficiency using a genetic algorithm. The genetic algorithm found sets of scaling

relations with high efficiencies, greater than a predefined limit. These sets were then sorted according to efficiency and the best ones were further optimized with the gradient method. The optimized scaling relations dictate the relationships that circuit elements in the model should have to obtain peak efficiencies. Circuit elements were found from these solutions by substituting typical values into the relations, producing a set of dimensional parameters (i.e.  $L_e$ ,  $C_1$ ,  $L_1$ , etc). Since several of these dimensional parameters are dependent on the coil and plasmoid geometry, a geometry needed to be fit to these values. This fitting process is largely trial-and-error, where different combinations of coil-plasmoid geometries are inserted into the dimensional form of the model (presented in Section III) and the governing equations (Eq. 1 - 7) are numerically integrated to find the resulting acceleration efficiency. It is not always possible to satisfy each parameter exactly, but if the acceleration efficiency predicted by the launcher model is close to (or better than) the efficiency found using the optimization techniques, it is considered a good solution. Details of the non-dimensionalization process, optimization routines, and evaluation of the dimensional model are detailed in the following sections.

### A. Non-Dimensional Model

The annular electromagnetic launcher model can be non-dimensionalized by expressing the independent variables and dependent variables as non-dimensional terms:  $t^* = t/t_0$ ,  $z^* = z/z_0$ ,  $u_z^* = u_z/u_0$ ,  $V^* = V/V_0$ ,  $I_1^* = I_1/V_0\sqrt{L_e/C_1}$ ,  $I_2^* = I_2/V_0\sqrt{L_e/C_1}$ , and  $I_3^* = I_3/V_0\sqrt{L_e/C_1}$ . Here  $u_0 = z_0/t_0$ . The variables  $t_0$  and  $z_0$  are time and length scales, which will be defined later. Substituting these into Equations 1 - 7 and forming groups reduces the system to the following non-dimensional equations:

$$\frac{du_z^*}{dt^*} = -\alpha \left( \frac{dk_{13}}{dz^*} \sqrt{\theta_1\theta_3} I_1^* I_3^* + \frac{dk_{23}}{dz^*} \sqrt{\theta_2\theta_3} I_2^* I_3^* \right) \quad (11)$$

$$\frac{dz^*}{dt^*} = u_z^* \quad (12)$$

$$V_c^* = \frac{1}{\tau_0} \left[ (\tau_0\gamma_e) (I_1^* + I_2^*) + (1 + \theta_1 + \theta_{e1}) \frac{dI_1^*}{dt^*} + \frac{dI_2^*}{dt^*} + \gamma_1\tau_0 I_1^* - k_{13}\sqrt{\theta_1\theta_3} \frac{dI_3^*}{dt^*} - \frac{dk_{13}}{dz^*} \sqrt{\theta_1\theta_3} I_3^* u_z^* \right] \quad (13)$$

$$0 = (\theta_1 + \theta_{e1}) \frac{dI_1^*}{dt^*} - (\theta_2 + \theta_{e2}) \frac{dI_2^*}{dt^*} + \gamma_1\tau_0 I_1^* - \gamma_2\tau_0 I_2^* + \left( k_{23}\sqrt{\theta_2\theta_3} - k_{13}\sqrt{\theta_1\theta_3} \right) \frac{dI_3^*}{dt^*} + \left( \frac{dk_{23}}{dz^*} \sqrt{\theta_2\theta_3} - \frac{dk_{13}}{dz^*} \sqrt{\theta_1\theta_3} \right) I_3^* u_z^* \quad (14)$$

$$0 = \theta_3 \frac{dI_3^*}{dt^*} + \gamma_p\tau_0 I_3^* - k_{13}\sqrt{\theta_1\theta_3} \frac{dI_1^*}{dt^*} - k_{23}\sqrt{\theta_2\theta_3} \frac{dI_2^*}{dt^*} - \left( \frac{dk_{13}}{dz^*} \sqrt{\theta_1\theta_3} I_1^* + \frac{dk_{23}}{dz^*} \sqrt{\theta_2\theta_3} I_2^* \right) u_z^* \quad (15)$$

$$\frac{dV_c^*}{dt^*} = -\tau_0 (I_1^* + I_2^*) \quad (16)$$

The new non-dimensional groups are defined as:

$$\gamma_e = R_e \sqrt{\frac{C_1}{L_e}} \quad \gamma_1 = R_1 \sqrt{\frac{C_1}{L_e}} \quad \gamma_2 = R_2 \sqrt{\frac{C_1}{L_e}} \quad \gamma_p = R_3 \sqrt{\frac{C_1}{L_e}} \quad (17)$$

$$\alpha = \frac{V_0^2 C_1}{m (z_0/t_0)^2} \quad \tau_0 = \frac{t_0}{\sqrt{L_e C_1}} \quad (18)$$

$$\theta_1 = \frac{L_1}{L_e} \quad \theta_2 = \frac{L_2}{L_e} \quad \theta_3 = \frac{L_3}{L_e} \quad \theta_{e1} = \frac{L_{e1}}{L_e} \quad \theta_{e2} = \frac{L_{e2}}{L_e} \quad (19)$$

Applying non-dimensional parameters  $\sigma_{13} = z_0/z_{13}$ ,  $\sigma_{23} = z_0/z_{23}$ ,  $\sigma_{s13} = z_{s13}/z_0$ , and  $\sigma_{s23} = z_{s23}/z_0$ , the coupling coefficients become:

$$k_{13}(z^*) = k_{13}^0 \exp(-(|z^* - \sigma_{s13}| \sigma_{13})^{n_{13}}) \quad (20)$$

$$k_{23}(z^*) = k_{23}^0 \exp(-(|z^* - \sigma_{s23}| \sigma_{23})^{n_{23}}) \quad (21)$$

where  $k_{13}^0$  and  $k_{23}^0$  are the nominal coupling coefficients for the outer coil-plasmoid and inner coil-plasmoid, respectively. The non-dimensionalization reduces the 22 original variables to 19 non-dimensional variables. Writing acceleration efficiency (Equation 10) in non-dimensional terms yields:

$$\eta_{accel}^* = \frac{(u_f^*)^2}{\alpha} \quad (22)$$

For this model, if the plasmoid cannot leave the coils during its lifetime it is considered to have a final velocity of zero and no acceleration efficiency. Setting  $z_0$  to the coil length and  $t_0$  as the confinement time, only parameters which satisfy the relation  $z^*(t^* = 1) \geq 1$  will be considered as solutions.

## B. Model Optimization: Genetic Algorithms and Gradient Based Methods

Genetic algorithms are a common technique for optimizing large parameter sets which contain many singularities, several local extrema, and non-differentiability. Since the annular electromagnetic launcher model has many parameter sets where the plasmoid fails to translate (singularities) and can have several maxima, a genetic algorithm is appropriate for this problem. Genetic algorithms use processes similar to genetic evolution to optimize a cost function (also called an objective function or fitness function). The algorithm starts by randomly selecting sets parameters. Each parameter is referred to as a chromosome. Each set of chromosomes form an individual and a group of individuals make up a generation. Each individual is then evaluated using a fitness function. In this case, the chromosomes for each individual are used as inputs for the non-dimensional form of the model and Equation 22 is used to evaluate the individual's fitness. The chromosomes from the "fittest" individual of the first generation are then used to form a second generation of individuals using methods referred to as reproduction, cross-over, and mutation. During reproduction, the individuals with the best fitness are given the highest probability of having their chromosomes replicated in the coming generation. Though less likely, some of the chromosomes from the least fit individuals can show up in the next generation to ensure a diverse population. This new population is then crossed and mutated. Cross-over is when two individuals swap single chromosomes and mutations occur where single chromosomes are randomly changed. This keeps the optimization routine from settling around a premature extrema. The second generation is evaluated for fitness and is reproduced, crossed, and mutated into a third generation. The process continues until either the generations stop evolving or some defined limit is reached.

A genetic algorithm using the principles outlined above was adapted from one made available for MatLab by Michael Gordy<sup>19</sup> and uses the methods of Dorsey and Mayer.<sup>20</sup>

Genetic optimization works well to locate the general vicinity of maxima for large parameter set problems. Gradient-based methods can then be used to pinpoint the extrema using the output from the genetic algorithm as a starting point. The ten highest individuals from several trials of the genetic algorithm were used as starting points for MatLab's multi-variable constrained optimization routine `fmincon`. This optimization tool begins at a starting point and calculates the derivative in each direction using either the finite difference method or a user-supplied gradient. The `fmincon` routine searches for a minimum (as opposed to a maximum) so the cost function used in the genetic algorithm had to be inverted. The minima of the inverted cost function is obtained when the change in the derivative falls below a set limit. The locations of extrema from the optimization routine are expressed in non-dimensional groups, implying an infinite combination of circuit elements are possible as long as they satisfy these relations. A single combination of circuit elements from this large pool can be found by substituting in values for  $z_0$ ,  $t_0$ , and one of the following circuit parameters:  $L_e$ ,  $L_1$ ,  $L_2$ ,  $L_3$ , or  $C_1$ . Since several of the circuit elements are geometry dependent (for example, coil inductance depends on coil geometry), coil and plasmoid dimensions must be fit to them. It must be cautioned that not all parameter combinations from the optimization routines have a physically viable solution. For example, to satisfy the required value for  $L_3$  might require a plasmoid which extends beyond the coils and this is not physically possible in an AFRC. To ensure that a good solution is found, the number of parameter sets selected from the output of the genetic algorithm should be sufficiently large (in this paper, ten sets were chosen) and diverse.



### C. Solving the Dimensional Model

A dimensional solution to the optimized scaling relations can be found by substituting in appropriate values for  $z_0$  (coil length),  $t_0$  (plasmoid lifetime), and a circuit element ( $L_e$ ,  $L_1$ ,  $L_2$ ,  $L_3$ , or  $C_1$ ) using the relations shown in Equation 17 - 19. This substitution provides a list of circuit elements  $C_1$ ,  $L_e$ ,  $L_{e1}$ ,  $L_{e2}$ ,  $R_e$ ,  $R_1$  and  $R_2$  which can be fixed at their resulting value. The remaining circuit elements are geometry dependent, so the remaining inner and outer coil geometries (radius and cone angle, coil length is equal on both coils and is fixed at  $z_0$ ) and plasmoid geometry (height, width) must be found so the corresponding inductances and coupling parameters roughly satisfy their expected value. This fitting process is largely done by trial-and-error. A good starting point for this process is to select coil dimensions and estimate their inductance using formularies, such as Lundin,<sup>21</sup> so they come close to  $L_1$  and  $L_2$ . These formularies are only valid for low frequency applications less than 1 kHz whereas the discharge frequency for AFRC devices is usually above 10 kHz, but they provide a good estimate.

Once coil dimensions are found so that estimates for  $L_1$  and  $L_2$  are close to their expected value, their frequency-dependent inductance can be calculated with electromagnetic field solvers. These field solvers can also be used to find the plasmoid inductance and the mutual inductance profiles between the coils and plasmoid. The electromagnetic field solvers chosen for this work are from COMSOL's Multiphysics software; a demonstration of how these solvers can be used to find inductance is provided here. In COMSOL, the coil and plasmoid geometry was modeled using a 2D-axisymmetry with a time-harmonic solver, insulative boundary conditions, and an arbitrary cone angle for the outer coil. A "plasmoid" was drawn to occupy a fraction of the space between the inner and outer coils, following the contours of the outer coil. The bounding geometry was made sufficiently large to minimize field disturbances around the coils and plasmoid. The mesh was generated using boundary layer meshing on the inner and outer coils to resolve skin currents and triangular meshing for the rest of the domain. Mesh elements no larger than 0.5 mm along the plasmoid boundary were used. This was the smallest element size that could be used without running out of memory and maintain sufficient resolution. Figure 4 displays a typical geometry with the mesh visible. The conductivity of the plasmoid was set to 30,000 S/m, found with Equation 9 ( $\sigma = 1/\rho$ ) and a plasma temperature of 10 eV. The self-inductance of each inductor  $L_1$ ,  $L_2$ , and  $L_3$  was found by making the other inductors invisible (conductivity of zero) and applying a sinusoidal loop potential ( $V_{loop} = 500$  V) to the remaining coil. The frequency for these simulations was calculated from  $C_1$  and the total circuit inductance, using the Lundin estimated values for  $L_1$  and  $L_2$  ( $\omega = 1/\sqrt{LC}$ ). Over a small range of frequencies, inductance does not change significantly so generally this estimated frequency was sufficient. Once the COMSOL simulations were completed, the frequency was recalculated with the new coil inductance values. If it differed from the original frequency by more than 50 percent, the inductance simulations were repeated with the new frequency. After proper boundary conditions, conductivity, frequency, and potentials were applied, the currents and fields were calculated with the electromagnetic solvers. The inductance was then found from inductive impedance:  $L = \text{imag}(V_{loop}/I_{tot}\omega)$ , where  $I_{tot}$  is the integrated current density (total current) in the coil and  $\omega$  is the frequency of the signal.

The mutual inductance between the coils and plasmoid  $M_{13}$ ,  $M_{23}$  was found by applying a loop potential  $V_{loop}$  to only one of the coils. The conductivity of the plasmoid was set to 30,000 S/m and the other coil was neglected (conductivity of zero). The currents and fields were found and the inductance on the coil was calculated from inductive impedance. The current induced in the plasmoid will change the current distribution in the coil, so the inductive impedance in the coil is actually a combination of self inductance of the inner or outer coil  $L_c$  (depending on which one was activated), plasmoid  $L_p$ , and the mutual inductance  $M$ . The three quantities can be related through the following equation:

$$L_{eff} = L_c - \frac{M^2}{L_p} \quad (23)$$

This equation can be verified through a quick circuit analysis and is cited elsewhere.<sup>1,14</sup> To find how the mutual inductance varies with plasmoid position, the plasmoid is moved to a new axial position and the analysis is repeated. A curve fit to the resulting data yields values for the coefficients in Equation 8.

The mutual inductance profile and coil inductances calculated using COMSOL can be supplied to the annular electromagnetic launcher model described by Equations 1 - 7, along with the specified circuit parameters  $C_1$ ,  $L_e$ ,  $L_{e1}$ ,  $L_{e2}$ ,  $R_e$ ,  $R_1$  and  $R_2$ . Plasmoid mass can be found using the volume of the plasmoid and the plasma density (typically close to  $10^{19}$  particles/m<sup>3</sup>). The charging voltage on the capacitor is calculated by inserting  $C_1$ ,  $z_0$ ,  $t_0$ , and  $m$  into the relation for  $\alpha$  (Eq. 18). Plasmoid resistance  $R_3$  is found from

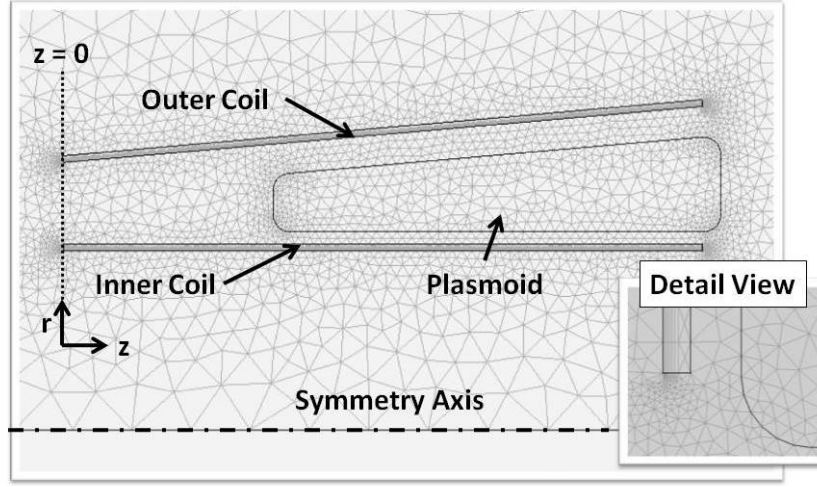


Figure 4. Typical coil mesh and geometry used for COMSOL simulations. The detail view shows the boundary layer mesh used on the coils to resolve skin currents.

plasmoid volume and Spitzer resistivity. The model equations with the inputs can numerically integrated to find the currents and plasmoid trajectories. In this work, MatLab's `ode45` numerical integration tool was used. It is an explicit 4th order Runge-Kutta solver, ideal for medium-accuracy, non-stiff problems. Once the plasmoid position and velocity are found, the acceleration efficiency  $\eta_{accel}$  for a plasmoid can be calculated using Equation 10. If the plasmoid's position at  $t = t_0$  is less than the coil length  $z_0$ , the plasmoid does not translate from the coils so  $\eta_{accel}$  is zero. If  $\eta_{accel}$  is close to ( $\pm 0.1$ ) or greater than the acceleration efficiency predicted by the optimization routines  $\eta_{accel}^*$ , the coil geometry and circuit elements are considered an acceptable solution. If  $\eta_{accel}$  falls short of  $\eta_{accel}^*$ , a new coil or plasmoid must be selected and the process re-iterated until an acceptable solution is reached.

#### D. Verification of Numerical Methods and Model

The methods for finding mutual inductance profiles using electromagnetic field solvers was verified by reconstructing the PTX experiment by Martin and Eskridge<sup>14</sup> in COMSOL's Multiphysics software. The PTX experiment used a conical theta pinch coil of  $17.5^\circ$  with an aluminum slug to mimic the plasmoid. They measured an effective inductance profile experimentally and calculated one using QuickField software. Repeating their experiment in COMSOL provided the  $L_{eff}(z)$  shown in Figure 5. Comparing this profile (labeled in Figure 5 as COMSOL data) to the results obtained by Martin and Eskridge shows good agreement in trend and minimal difference in magnitude. These findings substantiate the computational methods for determining the mutual inductance profile  $M(z)$  as outlined previously.

Since experimental studies of inductive annular electromagnetic launchers are not available, the annular electromagnetic launcher model was validated in two steps. First, the equations of motion (Equations 1, 2) coupled with the circuit equations (Equations 3, 4, 5, 6, 7) were tested using data published for the ring launcher by Novac.<sup>16</sup> The inner coil circuit was ignored by setting  $R_2 = 100 \Omega$ ,  $L_2$ ,  $M_{23} = 0$ . Setting a high impedance in this branch of this circuit shunts 99.99 percent of the current through the outer

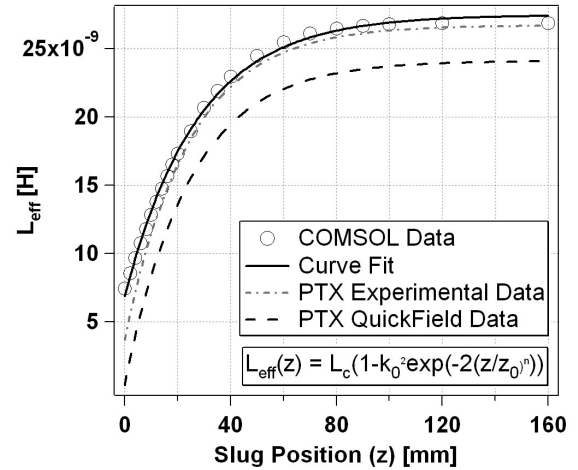


Figure 5. Comparison of effective inductance profiles for the PTX experiment.

coil circuit and the circuit behaves as if the inner coil were not there. The remaining parameters were set to values from Table 1 in Novac:<sup>16</sup>  $m = 0.7$  g,  $C_1 = 31.8$   $\mu$ F,  $L_e = 12.7$  nH,  $R_e = 1$  m $\Omega$ ,  $L_1 = 63.6$  nH, and  $L_3 = 74.7$  nH. The projectile resistance  $R_3$  was set to 15 m $\Omega$  and the capacitor charged to 30 kV. The mutual inductance profile was not specified, but Martin and Eskridge who performed a similar validation study for a model they developed used  $k_0 = 0.98$ ,  $z_0 = 5$  mm,  $n = 1$ , and  $z_s = 0$  with good results. The annular electromagnetic launcher model with these parameters determined the final projectile velocity to be 3.78 km/s, near the 4 km/s terminal velocity reported by Novac. This validation step was done for both the non-dimensional and dimensional form of the model.

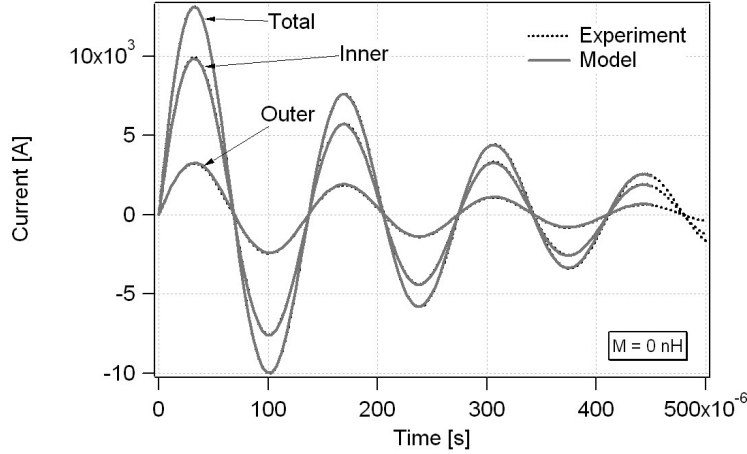


Figure 6. Comparison of circuit currents  $I_0$ ,  $I_1$ ,  $I_2$  with XOCOT-T experimental data.

Following validation of the equations of motion, the coil circuit equations (Equations 3, 4, 5, 7) were tested using circuit parameters from the XOCOT-T.<sup>4</sup> These include:  $V_c = 1444$  V,  $C = 225$   $\mu$ F,  $L_e = 1.53$   $\mu$ H,  $R_e = 16$  m $\Omega$ ,  $L_1 + L_{e1} = 2.29$   $\mu$ H,  $L_2 + L_{e2} = 746$   $\mu$ H,  $R_1, R_2 = 1$  m $\Omega$ . The coil, transmission line, and external inductances for the XOCOT-T were measured with an Agilent 4294A Impedance Analyzer. The external resistance  $R_{ext}$  was adjusted to match the damping on the experimental data, since this value was not measured. The circuit equations (Equations 3, 4, 5, 7) were solved to find  $I_0$ ,  $I_1$ , and  $I_2$ . These were compared to measurements from Pearson current monitors on the XOCOT-T experiment. As is shown in Figure 6, exceptional agreement between the experimental data and the circuit solutions was found. This, combined with the launcher model validation outlined in the previous paragraph, shows that the annular electromagnetic launcher model should provide fairly reliable results.

## V. Preliminary Results and Discussion

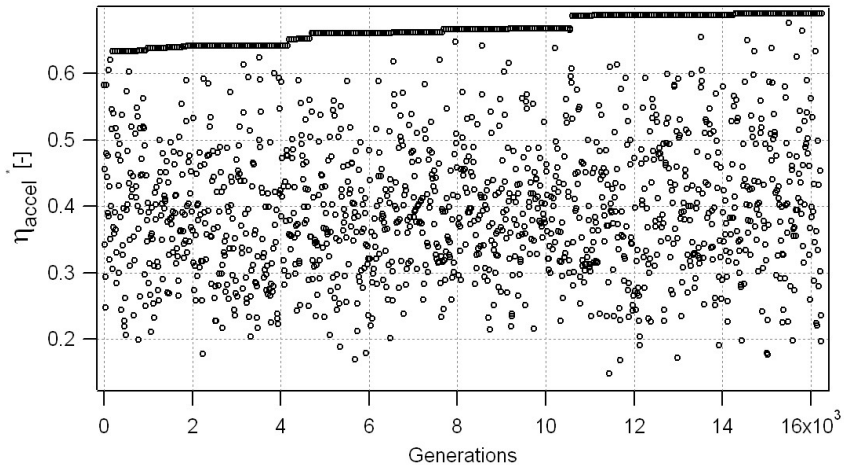
The optimization methodology detailed above was demonstrated by running a test case. Though the results are preliminary, they indicate that the methodology presented in this paper can result in a physically-realizable geometry. The test case was started by using the genetic algorithm routine written in MatLab to find parameter combinations with  $\eta_{accel}^* \geq 0.5$  for the non-dimensional form of the annular electromagnetic launcher model. Upper bounds and lower bounds for the non-dimensional parameter space were defined as shown in Table 1. An initial guess was also provided. This was used as the chromosome set for one individual of the first generation. The remaining individuals were randomly seeded.

Table 1 shows the default settings; three different trials were performed with the upper bound of  $\alpha$  set to 100, 500, and 5000. Each trial was started using a population size of 20 individuals per generation. Dorsey and Mayer suggest a population size of 24, however anything greater than 20 increased the total computation time extensively. The individual with the highest efficiency from each generation was stored until an individual with a higher fitness was found. To decrease the time to convergence, every 50 iterations a random individual was replaced by this fittest individual. The simulation was stopped when the efficiency did not increase by  $10^{-4}$  in 2000 consecutive generations. The algorithm created over 16,000 generations, taking 116 hours to complete. A depiction of how the generations evolved over the course of the simulation can be shown in Figure 7, where the best efficiency from every 10th generation is plotted. By following

**Table 1. Parameter Space Definitions for the Genetic Algorithm**

parameter	$\theta_1$	$\theta_2/\theta_1$	$\theta_3/\theta_1$	$\gamma_e$	$\gamma_p$	$\alpha$	$\tau_0$	$k_{13}$	$k_{23}$	$\sigma_{13}$	$\sigma_{23}/\sigma_{13}$
upper	10	0.5	0.5	1	1	1000	5	0.7	0.7	2.0	1.1
lower	0.001	0.001	0.001	$10^{-4}$	$10^{-4}$	0.001	0.001	0.1	0.1	0.1	0.9
initial	1.0	0.01	0.01	$10^{-4}$	$10^{-4}$	0.001	0.01	0.55	0.55	0.2	1.0
	$n_{13}$	$n_{23}$	$\theta_{e1}$	$\theta_{e2}$	$\gamma_1$	$\gamma_2$	$\sigma_{s13}$	$\sigma_{s23}$			
upper	3.0	3.0	1.0	1.0	1.0	1.0	0.01	0.01			
lower	1.0	1.0	0.001	0.001	$10^{-4}$	$10^{-4}$	$10^{-4}$	$10^{-4}$			
initial	1.0	2.5	0.001	0.001	$10^{-4}$	$10^{-4}$	$10^{-4}$	$10^{-4}$			

the highest markers for each generation, it can be noticed how total efficiency increases over time before the solution terminates at  $\eta_{accel}^* = 0.69$ . While the overall fitness of the entire population increases with generation, there are still many generations with  $\eta_{accel}^*$  significantly below the leading  $\eta_{accel}^*$  as evidenced by the abundance of markers in the  $\eta_{accel}^* = 0.2$  to  $\eta_{accel}^* = 0.5$  region. From this, it appears that the generations do not evolve as quickly as they could. Speculation for why this happens is still preliminary, but it is thought that this could be caused by a high mutation rate or it could be due to the highly irregular nature of the fitness function. Further investigations into why this occurs as well as methods for reducing the computation time are necessary.

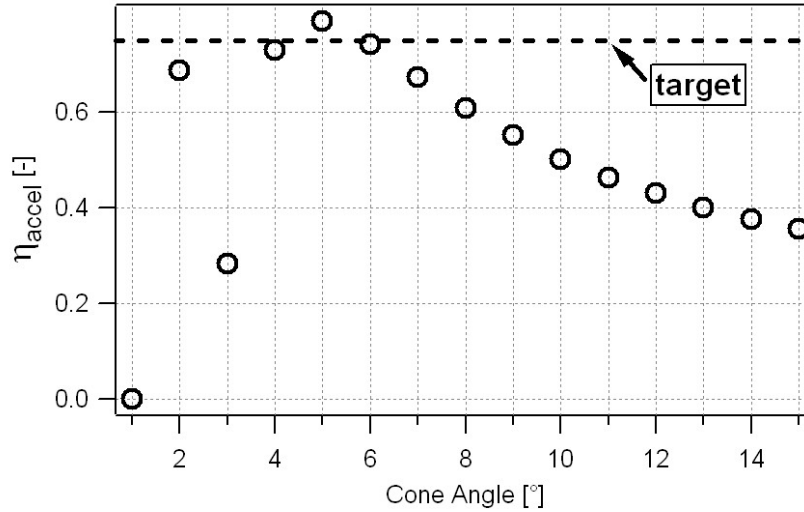


**Figure 7. Evolution of the genetic algorithm over 16,000 generations. The efficiency of each generation is plotted. Every 10th generation is shown for clarity.**

Each individual with an efficiency of greater than 0.5 was saved during the genetic algorithm routine. When the routine completed, these individuals were sorted according to efficiency and the ten fittest individuals from each trial were passed to the `fmincon` solver for the second round of optimization. For the trial shown in Table 1, all the top 10 individuals had an efficiency close to 0.69. They emerged from the second optimization round with efficiencies of 0.75, deviating from this by less than 0.0001. The small deviation indicates they are all fair solutions with regard to maximum efficiency; it is difficult to single out one solution based on efficiency. Not all the individuals generated during the second optimization round were physically viable solutions; though reasonable bounds were placed on the non-dimensional terms shown in Table 1, some of the results from the optimization routines were geometry combinations that are not physically possible. These were discarded and one individual out of the remaining individuals was chosen for further investigation. The non-dimensional parameter set from this individual is displayed in Table 2, as the "target" individual. For this individual,  $\eta_{accel}^* = 0.753$ .

The target individual's parameter set was put into dimensional terms by choosing the coil length and plasmoid lifetime:  $z_0 = 12$  inches and  $t_0 = 20 \mu s$ . The outer coil inductance  $L_1$  was also fixed by choosing

the coil to have a 5 inch radius and a single turn. From Lundin's formulary,<sup>21</sup> the inductance of this coil is approximately 152 nH. Using Equations 17 - 19 with the values shown in row 1 of Table 2, the following circuit elements were calculated:  $C_1 = 1,400 \mu\text{F}$ ,  $L_e = 15.2 \text{ nH}$ ,  $R_e, R_1, R_2 = 0.33 \mu\Omega$ , and  $L_{e1}, L_{e2} = 15.2 \text{ pH}$ . These inductances and resistances are lower than can probably be achieved in a physical circuit, since external inductances are usually 40 - 100 nH and resistances are typically slightly under 1 m $\Omega$ . As can be noticed from Table 2, peak efficiency happens when the scaling relations for these parameters hit their lower bound. Future studies should adjust the lower bounds on these parameters to a more appropriate level. Continuing with these parameters for demonstration purposes, a coil and plasmoid geometry must be found to satisfy the remaining target values. Unfortunately, this must be accomplished using trial-and-error. As a starting point, it was decided that the coil dimensions would be fixed and several different cone angles from  $1^\circ$  up to  $15^\circ$  would be tested. With the dimensions of the outer coil set to a radius of 5 inches and length of 12 inches, the inner coil should have a radius of 3.35 inches so that  $L_2 = 0.5L_1$  (see Table 2). The outer and inner coil were modeled in COMSOL and plasmoid was drawn between them to be 70 percent of the annulus width and annulus height. The coil inductances, plasmoid inductance, and mutual inductance profiles  $M_{13}(z)$  and  $M_{23}(z)$  were found from inductive impedance. Curve fits to the data for  $M_{13}(z)$  and  $M_{23}(z)$  yield the coefficients described in the equations. Since plasmoid resistance and plasmoid mass depend on plasmoid volume, these values were recalculated in-situ for each cone angle. Also, since  $\alpha = (C_1 V_c^2)/(m u_0^2)$ ,  $V_c$  also needed to be recalculated for each cone angle. As a sample, at a cone angle of  $5^\circ$ ,  $m = 2.25 \times 10^{-8} \text{ kg}$  (plasma density of  $3 \times 10^{19} \text{ m}^{-3}$ ),  $R_3 = 0.14 \text{ m}\Omega$  (conductivity of 30,000 S/m), and  $V_c = 1.0 \text{ kV}$ . The new values for  $L_1, L_2$ , and  $L_3$ , along with  $M_{13}$  and  $M_{23}$  were inserted into Equations 1 - 7. MatLab's `ode45` solver then calculated the coil currents and plasmoid trajectories at each cone angle. Acceleration efficiency was found for each plasmoid that was able to leave the coils during  $t_0$ . The results are plotted in Figure 8.



**Figure 8. Acceleration efficiency for different cone angles for dimensional study. The dashed line represents the target efficiency from the optimization routines.**

Acceleration efficiency depends strongly on the cone angle, as is apparent in Figure 8. This can be understood by examining Equation 1. Maximizing the kinetic energy of the plasmoid requires that the plasmoid reaches the axial location of peak  $dM/dz$  at the same time the coil currents peak. The peak  $dM/dz$  occurs right outside the ends of the coils, where the magnetic field diverges the greatest. Too small of a cone angle will keep the plasmoid from reaching this region during peak coil current and too large of a cone angle will eject the plasmoid before the peak currents develop. The cone angle that best matches the target efficiency is at  $5^\circ$  apparent from Figure 8. This solution has  $\eta_{\text{accel}} = 0.79$ .

Since the objective of this demonstration was to find scaling relations which result in the highest efficiency and see if a physical solution can be found to satisfy these relations through trial and error, it is useful to compare the target parameters with the physical solution. The circuit parameters used to obtain the results shown in Figure 8 at  $5^\circ$  (Case 5), were converted back into non-dimensional terms using the scaling relations shown in Equations 17 - 19. The comparison is shown in Table 2. Most of the candidate parameters differ

**Table 2.** Comparison of Parameter Set from Optimization Studies and Dimensional Studies

parameter	$\theta_1$	$\theta_2/\theta_1$	$\theta_3/\theta_1$	$\gamma_e$	$\gamma_p$	$\alpha$	$\tau_0$	$k_{13}$	$k_{23}$	$\sigma_{13}$	$\sigma_{23}/\sigma_{13}$
target	10	0.5	0.4944	$10^{-4}$	$10^{-4}$	273.04	4.3264	0.7	0.7	1.4429	1.0470
case 5	11.0	0.43	0.77	$10^{-4}$	0.04	273.04	4.3264	0.76	0.72	1.05	1.10
change	0.1	-0.15	0.56	0	407	0	0	0.08	0.03	-0.27	0.05

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	$n_{13}$	$n_{23}$	$\theta_{e1}$	$\theta_{e2}$	$\gamma_1$	$\gamma_2$	$\sigma_{s13}$	$\sigma_{s23}$
target	2.06	2.80	0.001	0.001	$10^{-4}$	$10^{-4}$	$7.64 \times 10^{-4}$	0.0046
case 5	2.21	3.04	0.001	0.001	$10^{-4}$	$10^{-4}$	0	0.13
change	0.07	0.09	0	0	0	0	-1	27.66

from the target parameters by less than 30 percent. Though plasmoid resistance  $\gamma_P$  and plasmoid inductance  $\theta_3$  from the dimensional simulation is significantly higher than the optimal found by the optimization study, it is not enough to decrease the acceleration efficiency. The relative plasma inductance  $\theta_3$  and coupling coefficients  $k_{13}$  and  $k_{23}$  for the dimensional simulation (candidate solution) exceed the limits placed on the optimization routine, as is shown in Table 2. For the optimization study, these limits were set as "soft" limits or regarded as limits that technology and experimental design probably could not exceed. With the dimensional study, it is shown that it is possible to slightly exceed these limits and when it happens, it results in a greater than expected efficiency.

The optimization routine found that the maximum efficiency happens when  $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ ,  $k_{13}$  and  $k_{23}$  approach the upper limit and  $\gamma_E$ ,  $\gamma_P$ ,  $\gamma_1$ ,  $\gamma_2$ ,  $\theta_{e1}$ , and  $\theta_{e2}$  are minimal. This result should come as no surprise. The first group of terms maximizes the amount of energy from the capacitor which can be transferred to the coils and the last group of terms remove energy from the circuit without providing any acceleration to the plasmoid. With confirmation of how these terms affect efficiency, it would be instructive to fix these terms at their appropriate limits and conduct further optimization studies on the remaining terms to see how they impact solution.

As previously stated, the purpose of this preliminary exercise was to demonstrate the application of an optimization routine to the annular electromagnetic launcher model. Though this exercise requires further refinement and investigation, the preliminary findings indicate that it is possible to obtain meaningful results from these methods and close to the target efficiency. Most of the values obtained from the dimensional study come within 30 percent of the target value. This is exciting when one considers following discussion. The AFRC setup is a dynamic system where the circuit elements and coil geometries strongly influence each other's behavior. For instance, the outer coil is an inductor and therefore contributes to the discharge frequency, which through the skin effect dictates how the currents are distributed in the coils. The current distribution will change the inductance of the coil, the inductance of the total system, and the coupling of the coils and plasmoid ( $k_{13}$ ,  $k_{23}$ ,  $z_{13}$ ,  $z_{23}$ ,  $z_{s13}$ ,  $z_{s23}$ ). Therefore, it is impossible to change one element of the setup without other elements changing. The optimization routine does not consider this dynamic behavior and independently selects parameters that will result in the best efficiency for the system. Once a final verdict is reached, a circuit and geometry must be fit to the outcome of the optimization routine. It is entirely likely that the parameters selected by the optimization will not have a physical solution (for example, the plasmoid might have to extend beyond the coils to achieve the target inductance). Having evidence that a viable solution can be reached promotes further development of this approach.

## VI. Conclusion

An annular electromagnetic launcher model has been developed to predict the translation physics of an AFRC plasmoid. This model has been optimized for peak efficiency using a combination of methods. Non-dimensionalization of the model provides scaling relations between model parameters to obtain a generalized solution. The non-dimensional form of the model was put into a genetic algorithm to find the relative location of local maxima. These parameter sets were then re-optimized using a gradient based solver (`fmincon` in MatLab) to pinpoint the maximum efficiency. A set of dimensional parameters was fit to one of the scaling

relation sets from the optimization round. The annular electromagnetic launcher model was then used to find the efficiency of this dimensional parameter set. The result of  $\eta_{accel} = 0.79$  closely matched the predicted value from the optimization round  $\eta_{accel}^* = 0.75$ . The preliminary results of these studies indicate that it is possible to find a coil and plasmoid geometry to obtain the peak efficiency predicted by the optimization techniques. However, further refinement of the results are necessary before accurate efficiency predictions can be made.

## VII. Future Work

The annular electromagnetic launcher model combined with the optimization routines presented in this paper is in the preliminary stage of investigation. Further work on this optimization can find ways to reduce the computation time and increase the number of useful data sets from the optimization process. Sensitivity studies on the results can be done to show how certain parameters contribute to the overall efficiency. This model is based on information about the AFRC plasmoid that is not readily available. Experiments on stationary plasmoids are necessary to get more information about plasma temperatures, densities, and plasmoid shapes. Finally, experimental studies on AFRCs are necessary to validate the model and provide information about AFRC plasmoid translation which is not treated in this model (plasmoid expansion, magnetic fields, etc).

## VIII. Acknowledgements

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